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THE APPLICATION OF J INTEGRAL TO MEASURE COHESIVE LAWS IN MATERIALS UNDERGOING LARGE SCALE YIELDING

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ABSTRACT

We explore the possibility of determining cohesive laws by the J-integral approach for materials having non-linear stress-strain behaviour (e.g. polymers and composites) by the use of a DCB sandwich specimen, consisting of stiff elastic beams bonded to the non-linear test material, loaded with pure bending moments. For a wide range of parameters of the non-linear material, the plastic unloading during crack extension is small, resulting in J integral values (fracture resistance) that deviate maximum 15% from the work of the cohesive traction. Thus the method can be used to extract the cohesive laws directly from experiments without any presumption about their shape. Finally, the DCB sandwich specimen was also analysed using the I integral to quantify the overestimation of the steady-state fracture resistance obtained using the J integral based method.

1 INTRODUCTION

Delamination (crack growth along an interface between layers) and splitting (crack growth parallel to the fibre direction) are important failure modes in composite materials. The failure modes are frequently modelled by the use of cohesive zone modelling. In cohesive zone modelling, the fracture process is modelled in terms of a traction-separation law called a cohesive law. A cohesive law describes the normal traction σ as a function of δ , the separation between the crack faces (the crack opening displacement), $\sigma(\delta)$.

The prediction of the load-carrying capacity of composite structures by the use of cohesive laws in numerical methods (e.g. the finite element method) has advanced significantly over the last decade. However, the accuracy of the predicted results depends heavily on the accuracy of the cohesive laws used in the simulations. Determination of cohesive law for real materials still remains a challenge. One approach is to derive cohesive laws from simultaneous measurement of the J integral and the end-opening of the cohesive zone, δ^* . The cohesive law can then be obtained by differentiation [1-2]:

$$\sigma(\delta^*) = \frac{\partial J}{\partial \delta^*} \quad (1)$$

This approach requires the determination of the J integral. A well-suited test specimen for J integral testing is the double cantilever beam (DCB) specimen loaded with pure bending moments. For this configuration, the J integral can be obtained in closed analytical form for linear-elastic materials, both homogenous and multi-layered materials [3-4]. However, the determination of cohesive laws for materials that possess significant non-linear stress-strain relations remains a big challenge. One approach is to obtain the cohesive law by so called "iterative guessing", i.e. by performing numerical simulation of the test to be analysed and vary cohesive law parameters until the predicted load-displacement relationship matches the measured load-displacement relationship [5-6].

However, using the iterative approach, it may be difficult to ensure that the energy dissipation due to large-scale plasticity does not included in the cohesive law determination, since the plasticity law may have a significant effect on the overall load-displacement relationship. It is therefore of

interest to develop a new approach in which the cohesive law for a non-linear material can be determined from experiments for which plasticity has only a small effect on the cohesive law determination.

In the present paper we explore the possibility of using the J integral approach, eq. (1), for cohesive law determination for a material that possess non-linear stress-strain relationship. The J integral is valid for non-linear material [7]. Furthermore, materials that have a non-linear elastic-plastic (i.e. non-reversible) stress-strain law can be considered being a non-linear elastic material if there is no material unloading in any material points [8]. A DCB specimen loaded with pure bending moments has the interesting feature that it will experience a monotonically increasing moment as the cohesive zone develops, reaching a maximum value of the applied moment M and thus a maximum J integral value when the cohesive zone is fully developed. During the subsequent crack growth, M will remain at its maximum value and the J integral will remain at a steady-state value, J_{ss} . Thus, even though large-scale yielding may occur in the beams behind the crack tip, there will be no overall unloading and thus no material unloading except close to the crack tip where high stresses are anticipated.

A problem is, unfortunately, that it may not be easy experimentally to assess whether the J integral approach may be valid. Therefore, in the present paper, we study which properties (stress-strain and cohesive laws) gives so low plastic unloading so that the J integral approach still enables accurate cohesive law determination. We also explore the use of the I integral as a tool to get an independent measurement of steady-state fracture resistance J_c : By comparing J_c (from the I integral) with J_{ss} obtained from the J integral approach, it becomes possible to assess if the steady-state J integral value J_{ss} is much in error. If it is not, it may still be possible to determine cohesive laws accurately by the J integral approach, eq. (1).

2 THEORY

2.1 Specimen geometry

The specimen we propose is a DCB (double cantilever beam) sandwich specimen consisting of a relative stiff elastic material (material #1) attached to the non-linear test material (material #2). The test material may undergo large-scale yielding, but if the major contribution to the J integral comes from the elastic beam, it is expected that the error in the J integral calculation will be small. The DCB sandwich specimen is loaded with pure bending moments. The geometry of the proposed test specimen is shown in Fig. 1.

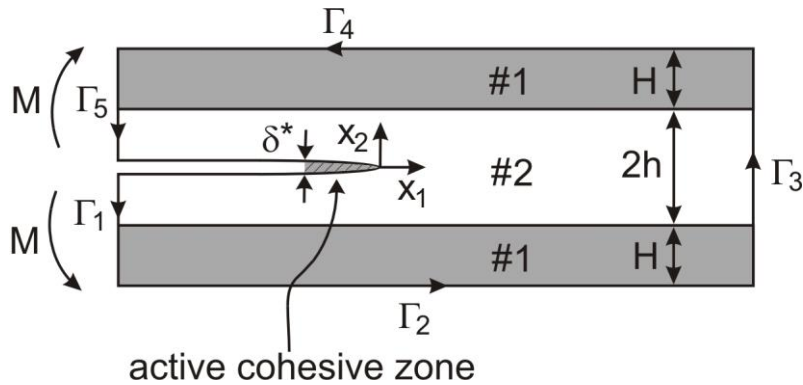


Figure 1: Geometry, loading and J integral paths for the DCB sandwich specimen.

2.2 Stress-strain laws

Material #1 (the beam material) is linear-elastic with a Young's modulus $E^{#1}$, while material #2 (the test material) is assumed to be non-linear elastic in such a way that the normal stress σ_{11} is related to the normal strain ε_{11} as

$$\sigma_{11}^{#1} = \begin{cases} E^{#2} \varepsilon_{11} - \sigma_0 (\varepsilon_{11})^2 & \text{for } 0 < \varepsilon_{11} < \varepsilon_u \\ \sigma_u & \text{for } \varepsilon_u < \varepsilon_{11} \end{cases} \quad (2)$$

where $E^{#2}$ is the Young's modulus of material #2 and σ_0 is a material constant, and ε_u is the strain value where σ_{11} attains its maximum value σ_u :

$$\sigma_u = \frac{(E^{#2})^2}{4\sigma_0} \quad \text{and} \quad \varepsilon_u = \frac{E^{#2}}{4\sigma_0} \quad (3)$$

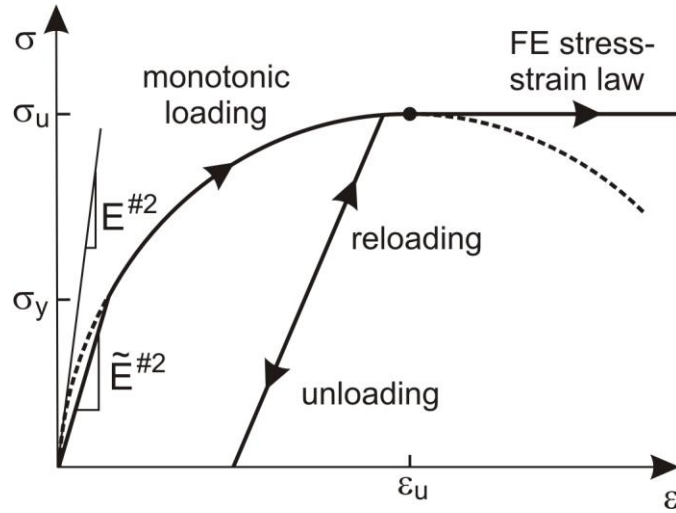


Figure 2: Schematics of the stress-strain for material #2.

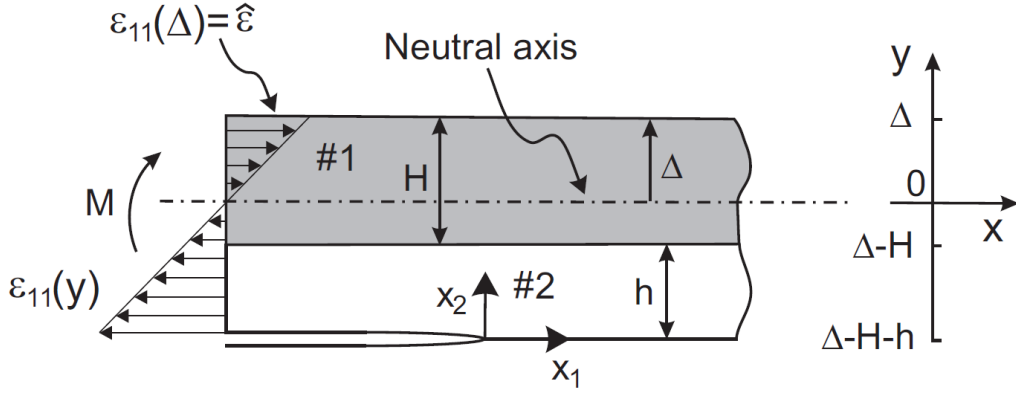
2.3 Sandwich beam parameters

The elastic centre (the distance Δ from the top of the beam to the x_2 - position where $\varepsilon_{11} = 0$ under pure bending), see Fig. 3, is calculated from elastic properties as

$$\frac{\Delta}{h} = \frac{1 + 2\bar{\Sigma}\eta + \bar{\Sigma}\eta^2}{2\eta(\bar{\Sigma}\eta + 1)} \quad (4)$$

where the non-dimensional parameters η and $\bar{\Sigma}$ are defined as

$$\eta = \frac{h}{H} \quad \bar{\Sigma} = \frac{E^{#2}}{E^{#1}} \quad (5)$$


 Figure 3: Position of the elastic center (neutral axis) and the x - y coordinate system.

2.4 J integral analysis

The J integral is evaluated along the external boundaries of the specimen. The only non-zero contributions come from the loaded ends below the cohesive zone where the beams are subjected to pure bending, i.e.,

$$\varepsilon_{11}(y) = \frac{\hat{\varepsilon}}{\Delta} y \quad \text{for } \Delta - H - h \leq y \leq \Delta \quad (6)$$

where y is a local coordinate system defined from the elastic center ($y = x_2 + \Delta - H - h$), see Fig. 3, and $\hat{\varepsilon}$ is the maximum strain at the top of the beam ($y = \Delta$). Only results are summarized here; details of the derivation is given elsewhere [9].

First, using (4) and (5), we calculate the non-dimensional parameters A and C from given elastic parameters:

$$A = \frac{(E^{\#2})^2}{16\Delta^2 H^2 E^{\#1} \sigma_u} \left[(\Delta - H)^4 - (\Delta - H - h)^4 \right] \quad (7)$$

and

$$C = \frac{E^{\#2}}{3\Delta H^2 E^{\#1}} \left[(\Delta - H)^3 - (\Delta - H - h)^3 \right] + \frac{1}{3\Delta H^2} \left[\Delta^3 - (\Delta - H)^3 \right] \quad (8)$$

Next we calculate a loading parameter, $D(M)$ which is a function of the applied moment, M :

$$D = -\frac{M}{BH^2 E^{\#1}} \quad (9)$$

The maximum strain in the beam $\hat{\varepsilon}(M)$ is found as a function of the moment M

$$\hat{\varepsilon} = \frac{-C + \sqrt{C^2 - 4AD}}{2A} \quad (10)$$

Finally, J_{ext} is calculated from [9]

$$J_{ext} = \frac{E^{#1} \hat{\varepsilon}^2}{\Delta^2} \left(\Delta^2 H - \Delta H^2 + \frac{H^3}{3} \right) + \frac{E^{#2} \hat{\varepsilon}^2}{\Delta^2} \left(\frac{h^3}{3} + h^2 (\Delta - H - h) + h (\Delta - H - h)^2 \right) + \frac{(E^{#2})^2 \hat{\varepsilon}^3}{3\sigma_0 \Delta^3} \left(\frac{h^4}{4} + h^3 (\Delta - H - h) + \frac{3h^2}{2} (\Delta - H - h)^2 + h (\Delta - H - h)^3 \right) \quad (11)$$

3 MODELLING

3.1 Finite element model

We now test the accuracy of the proposed analytical approach. We use a finite element (FE) model with a pre-defined trapezoidal cohesive law to generate simultaneous data of the applied moment M and the resulting end-opening δ^* . The cohesive law is specified in terms of the maximum traction value, $\hat{\sigma}$, which is reached at a characteristic separation δ_1 and remains until another characteristic separation δ_2 is reached. For separations beyond δ_2 , the traction decays linearly to zero at a critical separation denoted δ_c . The area under the traction-separation curve is the work per unit area of the cohesive traction, i.e. the fracture energy, J_c . The FE model uses the von Mises J_2 plasticity theory with isotropic hardening. The stress and strain fields and the development of the cohesive zone of the FE model are thus generated with full account to the development of crack tip plasticity including possible material unloading.

The output data from the FE model is analyzed using the J integral approach (i.e., implicitly assuming no material unloading) as if it was experimental data. The values of the J integral are obtained by eqs. (7)-(11), while the end-opening δ^* is obtained directly from the displacements computed for nodes at the end of the cohesive zone.

Having obtained J as a function of δ^* we proceed to obtain the cohesive law using (1). The differentiation is done without any data smoothening using the simple algorithm

$$\sigma_i = \frac{J_i - J_{i-1}}{\delta_i^* - \delta_{i-1}^*} \quad (12)$$

where i is the running number of data point sets (J and δ^*) of the particular FE simulation. The cohesive law obtained in this manner is then compared with the cohesive law originally specified as input in the FE simulation. In case the effect of local crack tip plasticity (and unloading) is insignificant, we anticipate the obtained cohesive law to be very similar to the specified cohesive law.

4 RESULTS

Fig. 4 shows three cohesive laws obtained from the FE results (solid lines), using the analytical analysis approach described above, compared with the specified cohesive laws (dashed lines) for $\hat{\sigma}/\sigma_u = 0.75$. The extracted cohesive laws are somewhat noisy (probably because the differentiation (12) is conducted using the "raw" data without any data smoothening). Never the less, the extracted cohesive laws all lie close to the specified cohesive laws. Cohesive law parameters such as $\hat{\sigma}$, δ_2 and δ_c can readily be identified.

Next, Fig. 5 shows extracted cohesive laws for different values of $\hat{\sigma}/\sigma_u$ close to unity. For a given separation value, the obtained traction value clearly lies above the specified traction value. With increasing $\hat{\sigma}/\sigma_u$, the traction value becomes slightly higher. In other words, with increasing $\hat{\sigma}/\sigma_u$, the obtained cohesive traction values are overestimated.

Fig. 6 shows the steady-state fracture resistance J_{ss} as a function of $\hat{\sigma}/\sigma_u$. If there were no plastic unloading at any points within the specimen, we would expect $J_{ss} = J_c$. Although the steady-state fracture resistance J_{ss} is higher than J_c , the difference is not really significant (less than 15%) for $\hat{\sigma}/\sigma_u < 1$. However, for $\hat{\sigma}/\sigma_u > 1$, J_{ss} increases rapidly with increasing $\hat{\sigma}/\sigma_u$.

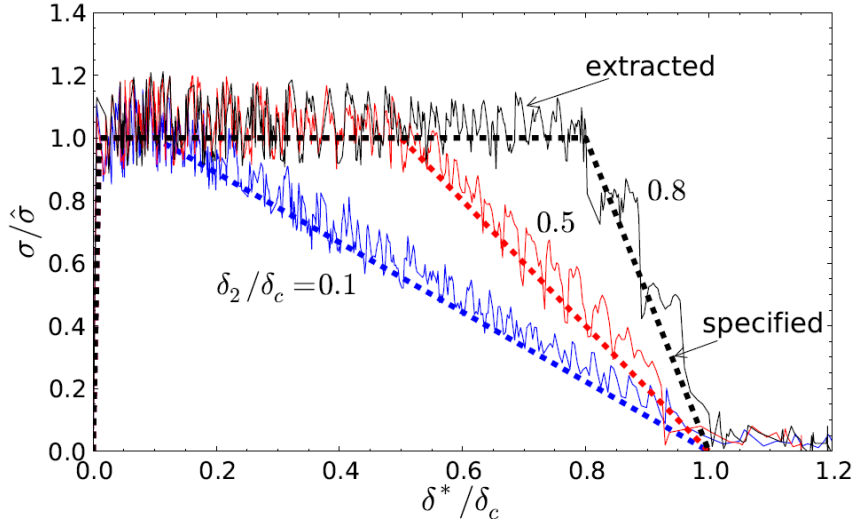


Figure 4: Comparison of original specified cohesive laws (dashed lines) and cohesive laws obtained using the J integral approach for $\hat{\sigma}/\sigma_u = 0.75$.

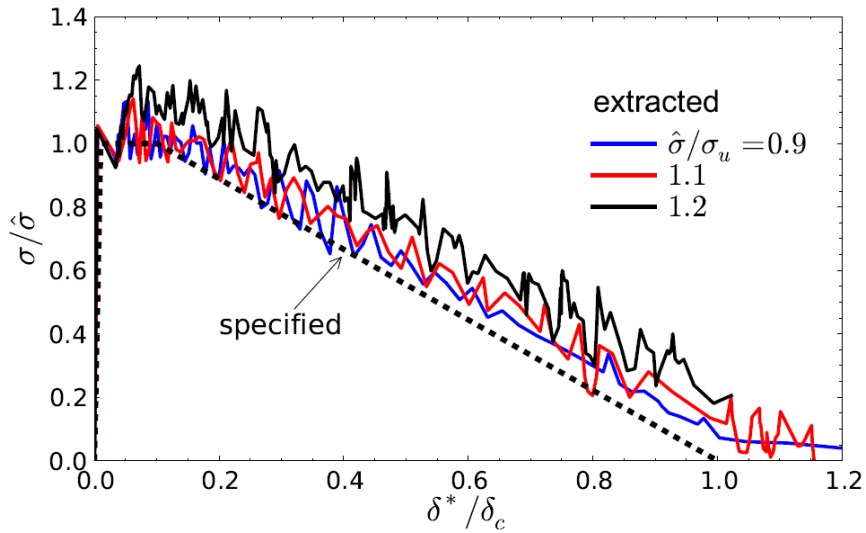


Figure 5: Comparison of original specified cohesive laws (dashed line) and cohesive laws obtained using the J integral approach for various values of $\hat{\sigma}/\sigma_u$.

5 DISCUSSION

5.1 Over-estimation of the steady-state fracture energy

The rapid increase in J_{ss} for $\hat{\sigma}/\sigma_u > 1$ can be understood as follows. For $\hat{\sigma}/\sigma_u \ll 1$, there is limited plasticity and thus the effect of non-elastic unloading at the crack tip is small. With increasing $\hat{\sigma}/\sigma_u$, the amount of crack tip plasticity increases. $\hat{\sigma}/\sigma_u > 1$ implies that the value of the peak cohesive traction is higher than the maximum stress that the plastic material can sustain. Then, material #2 will undergo yielding without crack growth.

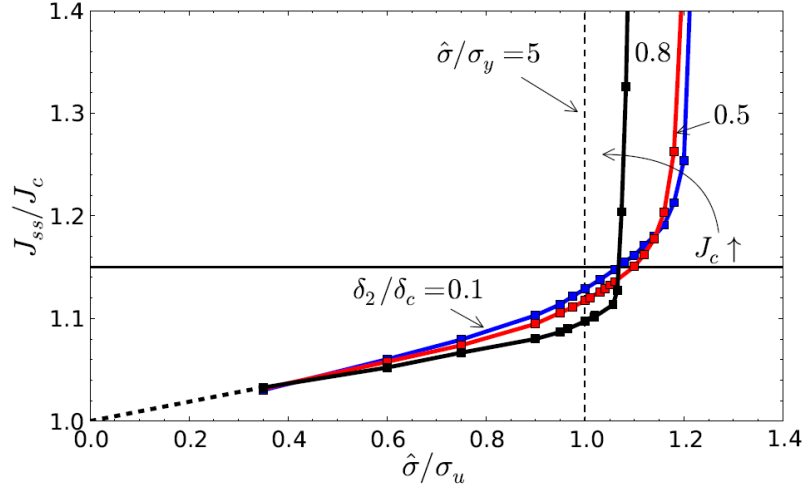


Figure 6: The steady-state fracture resistance J_{ss} as a function of $\hat{\sigma}/\sigma_u$ for three values of δ_2/δ_c .

5.2 Assessment of the steady-state fracture energy by the I integral

The J integral approach for determination of cohesive laws, eq. (1), seems to be fairly accurate as long as $\hat{\sigma}/\sigma_u < 1$. It is clear from Fig. 5 that for $\hat{\sigma}$ close to σ_u , it becomes difficult to assess whether the obtained cohesive law is correct. It would be useful to have an independent way of checking the accuracy of the $J - \delta^*$ data. Therefore, in the following we apply the I integral [10-11] to calculate the work of the cohesive traction by excluding the work of plasticity. The I integral approach is valid only under steady-state cracking. Under steady-state cracking, the I integral is path-independent (steady-state cracking implies that the stress states far ahead and far below the active cohesive zone are uniform (independent of x_1 -position) and that the stress field around the active cohesive law translate along the specimen in a self-similar fashion). The latter holds true since the DCB loaded with pure bending moments is a steady-state specimen - this does not hold true for some of the more conventional LEFM (linear-elastic fracture mechanics) test specimens [2]. Then, the evolution of strain of each material point at a given x_2 -position undergoes precisely the same history when the fracture process zone passes by the material point.

We analyze two integration paths. One is a path Γ_{loc} runs locally around the active cohesive zone while the other path, Γ_{ext} , that runs far ahead of the active cohesive zone, along the external boundaries, crossing across the beams at an x_1 -position behind the active cohesive zone but not deeper than the material points have undergone unloading to a steady-state stress state (similar to the J integral path shown in Fig. 1).

Evaluating the I integral along Γ_{loc} gives

$$I_{loc} = \int_0^{\delta_c} \sigma(\delta) d\delta = J_c \quad (13)$$

whereas an evaluation of the I integral along Γ_{ext} gives

$$I_{ext} = 2 \int_{\Delta}^{\Delta-H-h} \left(U(x_2, history) - \sigma_{il} \frac{\partial u_i}{\partial x_l} \right) dx_2 \quad (14)$$

The factor of two in front of the integral is to account for the fact that the integral includes only one beam; the result for the full specimen (two beams) is obtained by multiplying the result for one beam by a factor of two. The stress work function in (14) is given as

$$U(x_2, history) = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad (15)$$

The history dependence should be understood as follows: The stress-strain history should be traced for each material point, i.e. for all x_2 -values within the integration limits of (14), as the cohesive zone passes by (or conversely, as the material point moves by the cohesive zone). It should be noted that in the use of the I integral, no assumption are made with respect to nature of the plasticity law except that it must be time-independent; non-proportional loading and unloading are permissible.

In the present study we calculate I_{ext} numerically from results from the FE solutions for the fully developed cohesive law where the cracking occurs in steady-state. Due to path-independence, we expect the value of I_{ext} to be identical to I_{loc} , i.e., very close to J_c .

A few results for I_{ext} are superimposed in Fig. 7. It is seen that for the same value of $\hat{\sigma}/\sigma_u$, the I_{ext} results are much closer to J_c than the results for J_{ss} . The difference between I_{ext} and J_c is less than 4 % for the value of $\hat{\sigma}/\sigma_u$ for which J_{ss} deviates more than 15% from J_c . This suggests that it is possible to assess the magnitude of J_c by the use of the I integral. It should thus be possible to use the I integral as an independent consistency check of J_c obtained by the use of the J integral. More specifically, during steady-state crack, the displacement field and thus the history of the strain field can be assessed from images of the crack tip region by the use of the digital image correlation method (DIC), and then, by the use of the stress-strain law, can be assessed by a procedure similar to the one under for the data from the finite element simulations. Now in case the deviation between I_{ext} and J_{ss} is significant (e.g. exceeding 15%) we know then that the J integral approach for the determination of cohesive laws is invalid.

6 CONCLUSIONS

A sandwich specimen, created by attaching relative stiff elastic beams to a non-linear elastic (or elastic-plastic) test material, can be used to determine cohesive laws using a J integral approach in which the cohesive traction is obtained fairly accurately by differentiation of J with respect to the end-opening δ^* even though large scale yielding occurs in the test material. The I integral can be used as an independent means to assess the steady-state fracture resistance J_c . In case the difference between the results for I_{ext} and J_{ss} is small, the J integral approach will give accurate cohesive law measurements.

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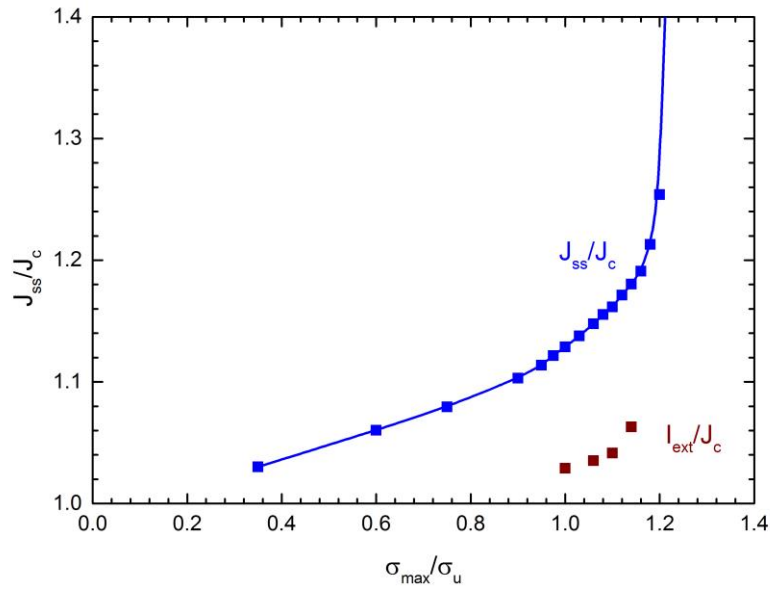


Figure 7: The steady-state fracture resistance J_{ss} as a function of $\hat{\sigma}/\sigma_u$ for values of $\delta_2/\delta_c = 0.1$. Results of for the I integral are superimposed.

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